

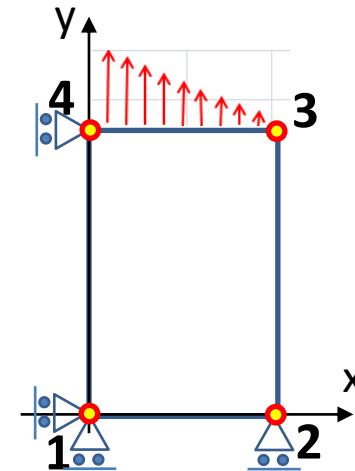
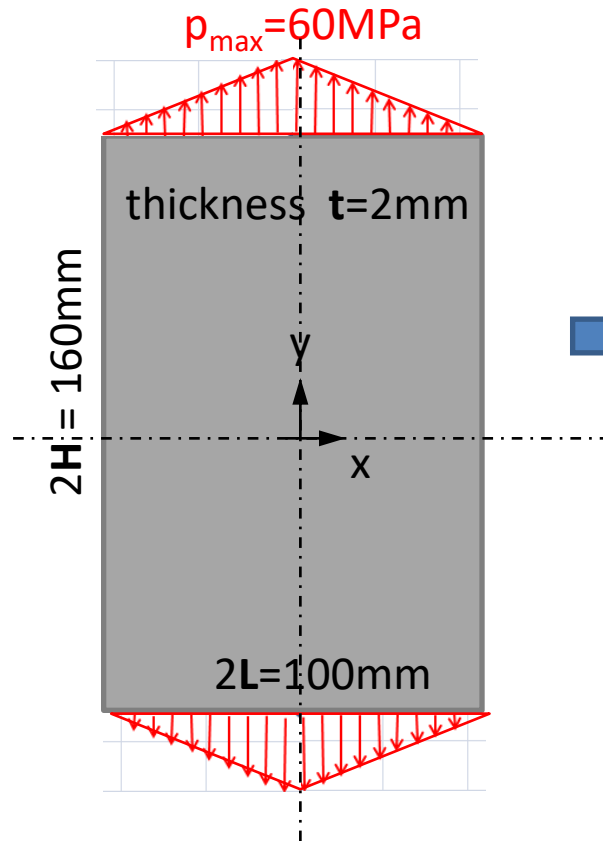


Finite element method (FEM1)

Lecture 6C. Plate 2D modeled with 4-node elements

03.2025

Example. 2D plate modeled using 4-node element



one-quarter model with
single 4-node element

vectors of nodal coordinates:

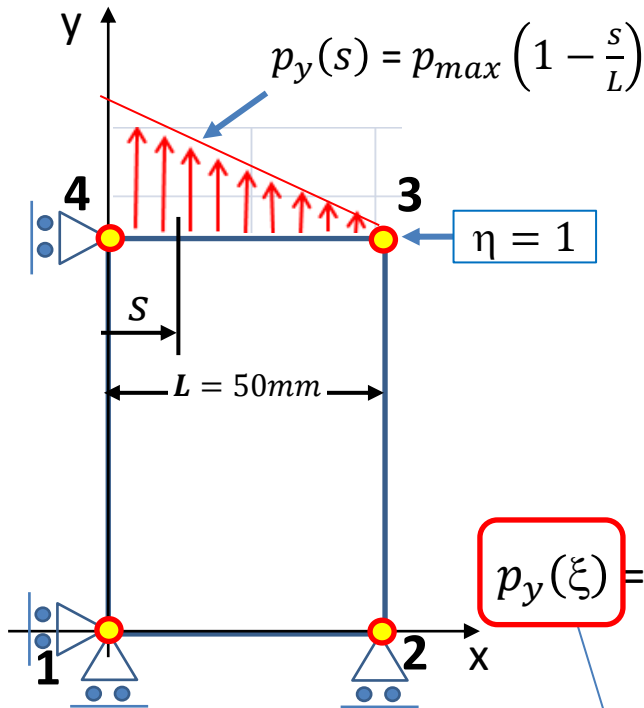
$$[x_i]_1 = [0, L, L, 0]$$

$$[y_i]_1 = [0, 0, H, H]$$

Equivalent load vector due to surface forces

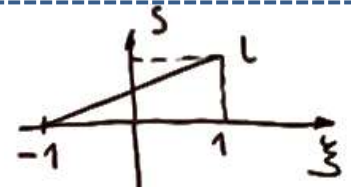
$$[F^p]_e = t_e \int_{-1}^1 [p_x, p_y] [N] \sqrt{\left(\frac{\partial [N(\xi,1)]}{\partial \xi} \{x_i\}_e \right)^2 + \left(\frac{\partial [N(\xi,1)]}{\partial \xi} \{y_i\}_e \right)^2} d\xi$$

1×8 2×8 1×4 4×1 1×4 4×1



$$\begin{aligned} x_3 &= L, & x_4 &= 0 \\ y_3 &= H, & y_4 &= H \end{aligned}$$

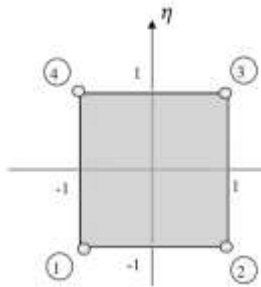
$$\left. \begin{aligned} s=0 &\rightarrow \xi = -1 \\ s=L &\rightarrow \xi = +1 \end{aligned} \right\} \rightarrow s(\xi) = \frac{L}{2}(\xi + 1)$$



$$[F^p]_1 = t \int_{-1}^1 [0, p_y(\xi)] [N] \sqrt{\left(\frac{\partial [N(\xi,1)]}{\partial \xi} \{x_i\}_e \right)^2 + \left(\frac{\partial [N(\xi,1)]}{\partial \xi} \{y_i\}_e \right)^2} d\xi$$

1×8 2×8 1×4 4×1 1×4 4×1

natural coordinate system

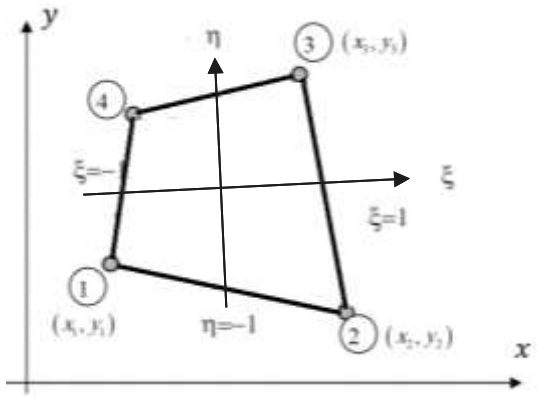


shape functions of the element and their derivatives in the natural system:

$$\begin{aligned} N_1(\xi, \eta) &= \frac{1}{4}(1 - \xi)(1 - \eta) \\ N_2(\xi, \eta) &= \frac{1}{4}(1 + \xi)(1 - \eta) \\ N_3(\xi, \eta) &= \frac{1}{4}(1 + \xi)(1 + \eta) \\ N_4(\xi, \eta) &= \frac{1}{4}(1 - \xi)(1 + \eta) \end{aligned}$$

i	$N_i(\xi, 1)$
1	0
2	0
3	$\frac{1}{2}(1 + \xi)$
4	$-\frac{1}{2}(1 - \xi)$

cartesian coordinate system



i	$\frac{\partial N_i}{\partial \xi}$	$\frac{\partial N_i}{\partial \eta}$
1	$-\frac{1}{4}(1 - \eta)$	$-\frac{1}{4}(1 - \xi)$
2	$\frac{1}{4}(1 - \eta)$	$-\frac{1}{4}(1 + \xi)$
3	$\frac{1}{4}(1 + \eta)$	$\frac{1}{4}(1 + \xi)$
4	$-\frac{1}{4}(1 + \eta)$	$\frac{1}{4}(1 - \xi)$



$\frac{\partial N_i(\xi, 1)}{\partial \xi}$
$-\frac{1}{4}(1 - 1) = 0$
$\frac{1}{4}(1 - 1) = 0$
$\frac{1}{4}(1 + 1) = \frac{1}{2}$
$-\frac{1}{4}(1 + 1) = -\frac{1}{2}$

$$\begin{aligned} x_3 &= L, \quad x_4 = 0 \\ y_3 &= H, \quad y_4 = H \end{aligned}$$

$$\begin{aligned} \frac{\partial N_3(\xi, 1)}{\partial \xi} &= 1/2, \quad \frac{\partial N_4(\xi, 1)}{\partial \xi} = -1/2 \\ N_3(\xi, 1) &= 1/2(1 + \xi), \quad N_4(\xi, 1) = -1/2(1 - \xi) \end{aligned}$$

$$[F^p]_1 = t \int_{-1}^1 [0, p_y(\xi)] [N] \sqrt{\left(\frac{\partial [N(\xi, 1)]}{\partial \xi} \{x_i\}_e \right)^2 + \left(\frac{\partial [N(\xi, 1)]}{\partial \xi} \{y_i\}_e \right)^2} d\xi$$

$$\frac{\partial [N(\xi, 1)]}{\partial \xi} \{x_i\}_1 = \frac{\partial N_3(\xi, 1)}{\partial \xi} x_3 + \frac{\partial N_4(\xi, 1)}{\partial \xi} x_4 = 1/2 \cdot L - 1/2 \cdot 0 = \frac{L}{2}$$

$$\frac{\partial [N(\xi, 1)]}{\partial \xi} \{y_i\}_1 = \frac{\partial N_3(\xi, 1)}{\partial \xi} y_3 + \frac{\partial N_4(\xi, 1)}{\partial \xi} y_4 = 1/2 \cdot H - 1/2 \cdot H = 0$$

$$\sqrt{\left(\frac{\partial [N(\xi, 1)]}{\partial \xi} \{x_i\}_e \right)^2 + \left(\frac{\partial [N(\xi, 1)]}{\partial \xi} \{y_i\}_e \right)^2} = \frac{L}{2}$$

$$[F^p]_1 = t \frac{L}{2} \int_{-1}^1 [0, p_y(\xi)] [N(\xi, 1)] d\xi$$

$$[N(\xi, 1)] = \begin{bmatrix} 0 & 0 & 0 & 0 & N_3(\xi, 1) & 0 & N_4(\xi, 1) & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3(\xi, 1) & 0 & N_4(\xi, 1) \end{bmatrix}$$

$$N_3(\xi, 1) = \frac{1}{2}(1 + \xi) \quad , \quad N_4(\xi, 1) = -\frac{1}{2}(1 - \xi)$$

$$[N(\xi, 1)] = \begin{bmatrix} 0 & 0 & 0 & 0 & N_3(\xi, 1) & 0 & N_4(\xi, 1) & 0 \\ 0 & 0 & 0 & 0 & 0 & N_3(\xi, 1) & 0 & N_4(\xi, 1) \end{bmatrix}$$

$$[F^p]_1 = t \frac{L}{2} \int_{-1}^1 [0, p_y(\xi)] [N(\xi, 1)] d\xi$$

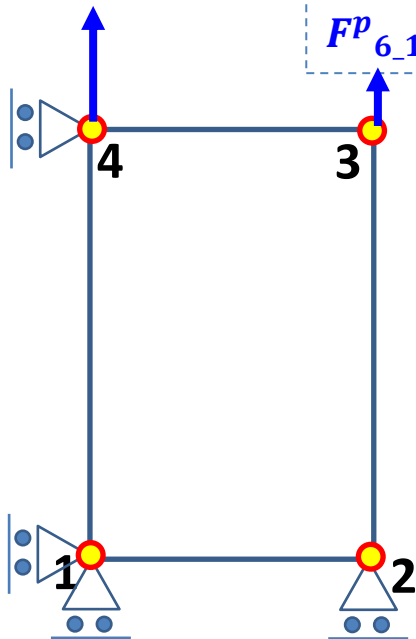
$$p_y(\xi) = \frac{p_{max}}{2} (1 - \xi)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & N_3(\xi, 1) & 0 & N_4(\xi, 1) \\ 0 & 0 & 0 & 0 & 0 & N_3(\xi, 1) & 0 & N_4(\xi, 1) \end{bmatrix}$$

$$[0, p_y(\xi)] \quad [0, 0, 0, 0, 0, \underbrace{N_3(\xi, 1)p_y(\xi)}_{F^p_{6-1}}, 0, \underbrace{N_4(\xi, 1)p_y(\xi)}_{F^p_{8-1}}]$$

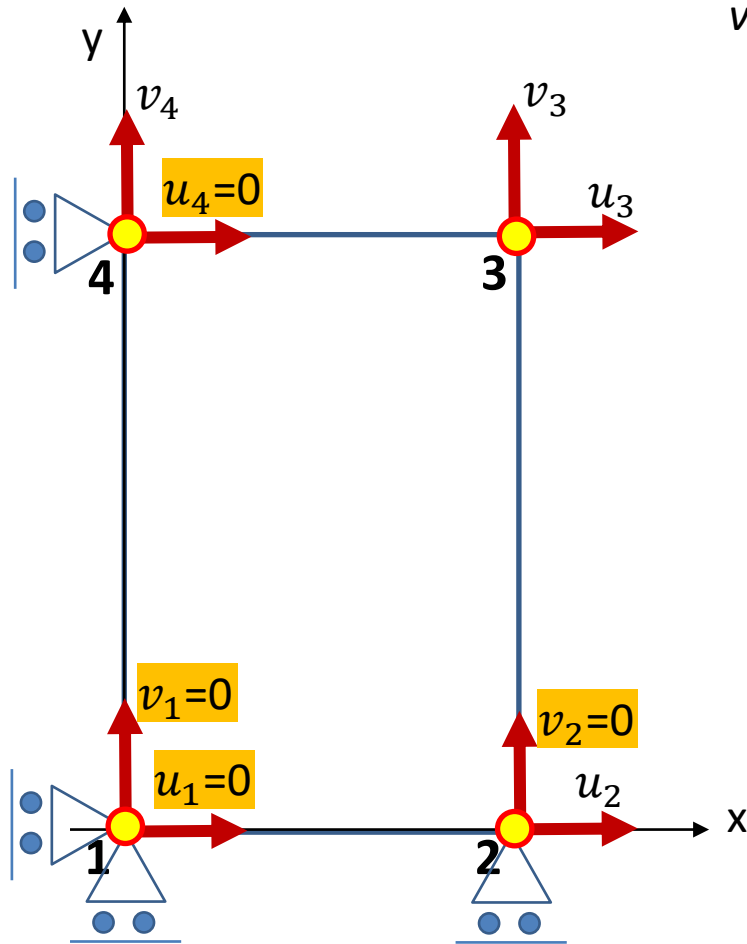
$$F^p_{8-1} = \frac{tL}{3} p_{max}$$

$$F^p_{6-1} = \frac{tL}{6} p_{max}$$



$$F^p_{6-1} = t \frac{L}{2} \int_{-1}^1 \frac{p_{max}}{2} (1 - \xi) \frac{1}{2}(1 + \xi) d\xi = \frac{tL}{6} p_{max}$$

$$F^p_{8-1} = t \frac{L}{2} \int_{-1}^1 \frac{p_{max}}{2} (1 - \xi) \left(-\frac{1}{2}(1 - \xi)\right) d\xi = \frac{tL}{3} p_{max}$$



vector of element nodal parameters

$$\{q\}_1 = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}_1 \quad \text{column vector}$$

8×1

$$[q]_1 = [u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4]_1$$

1×8

row vector

Boundary conditions: $N = 8 - 4 = 4$

vector of active nodal parameters after taking into account boundary conditions :

$$[q]_1 = [u_2, u_3, v_3, v_4]_1$$

1×4

Gradient matrix for the plain stress condition:

$$\begin{aligned}
 [R]_{3 \times 2} &= \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \frac{\partial}{\partial \xi} - \frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \frac{\partial}{\partial \eta} \right) & 0 \\ 0 & \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \frac{\partial}{\partial \eta} - \frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \frac{\partial}{\partial \xi} \right) \\ \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \frac{\partial}{\partial \eta} - \frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \frac{\partial}{\partial \xi} \right) & \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \frac{\partial}{\partial \xi} - \frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \frac{\partial}{\partial \eta} \right) \end{bmatrix} = \\
 &= \begin{bmatrix} \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \frac{\partial}{\partial \xi} \right) & 0 \\ 0 & \left(-\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \frac{\partial}{\partial \xi} \right) \\ \left(-\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \frac{\partial}{\partial \xi} \right) & \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \frac{\partial}{\partial \xi} \right) \end{bmatrix} + \begin{bmatrix} \left(-\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \frac{\partial}{\partial \eta} \right) & 0 \\ 0 & \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \frac{\partial}{\partial \eta} \right) \\ \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \frac{\partial}{\partial \eta} \right) & \left(-\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \frac{\partial}{\partial \eta} \right) \end{bmatrix} = \\
 &= \begin{bmatrix} \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \right) & 0 \\ 0 & \left(-\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \right) \\ \left(-\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \right) & \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \right) \end{bmatrix} \frac{\partial}{\partial \xi} + \begin{bmatrix} \left(-\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \right) & 0 \\ 0 & \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \right) \\ \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \right) & \left(-\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \right) \end{bmatrix} \frac{\partial}{\partial \eta}
 \end{aligned}$$

strain-displacement matrix

$$[B]_{3 \times 16} = [R]_{3 \times 2} [N]_{2 \times 16} = \begin{bmatrix} \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \right) & 0 \\ 0 & \left(-\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \right) \\ \left(-\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} \right) & \left(\frac{1}{\det[J]} \frac{\partial y}{\partial \eta} \right) \end{bmatrix} \begin{bmatrix} \frac{\partial N}{\partial \xi} \end{bmatrix} + \begin{bmatrix} \left(-\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \right) & 0 \\ 0 & \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \right) \\ \left(\frac{1}{\det[J]} \frac{\partial x}{\partial \xi} \right) & \left(-\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} \right) \end{bmatrix} \begin{bmatrix} \frac{\partial N}{\partial \eta} \end{bmatrix}$$

$$\begin{aligned}
\det[J] &= \det \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial y}{\partial \xi} \frac{\partial x}{\partial \eta} = \\
&= \frac{\partial([N(\xi, \eta)]\{x_i\}_e)}{\partial \xi} \frac{\partial([N(\xi, \eta)]\{y_i\}_e)}{\partial \eta} - \frac{\partial([N(\xi, \eta)]\{y_i\}_e)}{\partial \xi} \frac{\partial([N(\xi, \eta)]\{x_i\}_e)}{\partial \eta} = \\
&= \left(\frac{\partial[N(\xi, \eta)]}{\partial \xi} \{x_i\}_e \right) \left(\frac{\partial[N(\xi, \eta)]}{\partial \eta} \{y_i\}_e \right) - \left(\frac{\partial[N(\xi, \eta)]}{\partial \xi} \{y_i\}_e \right) \left(\frac{\partial[N(\xi, \eta)]}{\partial \eta} \{x_i\}_e \right)
\end{aligned}$$

$\begin{matrix} 1 \times 4 & 4 \times 1 & 1 \times 4 & 4 \times 1 & 1 \times 4 & 4 \times 1 & 1 \times 4 & 4 \times 1 \end{matrix}$

$$\begin{aligned}
\frac{\partial \xi}{\partial x} &= \frac{1}{\det[J]} \frac{\partial y}{\partial \eta} = \frac{1}{\det[J]} \frac{\partial([N(\xi, \eta)]\{y_i\}_e)}{\partial \eta} = \frac{1}{\det[J]} \frac{\partial[N(\xi, \eta)]}{\partial \eta} \{y_i\}_e \\
\frac{\partial \eta}{\partial x} &= -\frac{1}{\det[J]} \frac{\partial y}{\partial \xi} = -\frac{1}{\det[J]} \frac{\partial([N(\xi, \eta)]\{y_i\}_e)}{\partial \xi} = -\frac{1}{\det[J]} \frac{\partial[N(\xi, \eta)]}{\partial \xi} \{y_i\}_e \\
\frac{\partial \xi}{\partial y} &= -\frac{1}{\det[J]} \frac{\partial x}{\partial \eta} = -\frac{1}{\det[J]} \frac{\partial([N(\xi, \eta)]\{x_i\}_e)}{\partial \eta} = -\frac{1}{\det[J]} \frac{\partial[N(\xi, \eta)]}{\partial \eta} \{x_i\}_e \\
\frac{\partial \eta}{\partial y} &= \frac{1}{\det[J]} \frac{\partial x}{\partial \xi} = \frac{1}{\det[J]} \frac{\partial([N(\xi, \eta)]\{x_i\}_e)}{\partial \xi} = \frac{1}{\det[J]} \frac{\partial[N(\xi, \eta)]}{\partial \xi} \{x_i\}_e
\end{aligned}$$

$\begin{matrix} 1 \times 4 & 4 \times 1 & 1 \times 4 & 4 \times 1 & 1 \times 4 & 4 \times 1 & 1 \times 4 & 4 \times 1 \end{matrix}$

$$\left[\frac{\partial N}{\partial \xi} \right]_{2 \times 8} = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} & 0 \\ 0 & \frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \frac{\partial N_3}{\partial \xi} & 0 & \frac{\partial N_4}{\partial \xi} \end{bmatrix}$$

$$\left[\frac{\partial N}{\partial \eta} \right]_{2 \times 8} = \begin{bmatrix} \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} & 0 \\ 0 & \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & 0 & \frac{\partial N_3}{\partial \eta} & 0 & \frac{\partial N_4}{\partial \eta} \end{bmatrix}$$

$$[D]_{3 \times 3} = \frac{E}{(1 - \nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1 - \nu) \end{bmatrix}$$

constitutive matrix for plain stress

element stiffness matrix:

$$[k]_e = t_e \int_{-1}^1 \int_{-1}^1 [B(\xi, \eta)]_{8 \times 3}^T [D]_{3 \times 3} [B(\xi, \eta)]_{3 \times 8} \det[J] d\xi d\eta$$

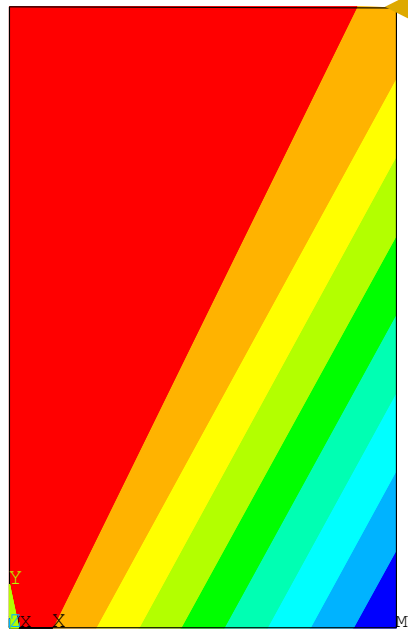
*numerical integration by the mixed quadrature rule
to avoid „shear locking“ (Enhanced strain formulation)*

$$\{q\} = [k]^{-1} \{F\}$$

4×1 4×4 4×1

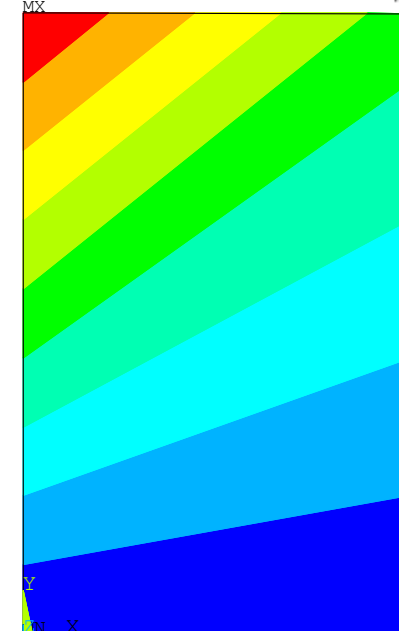
F		q	
0		.012724	u2
0		-.001562	u3
1000	N	.02286	v3
2000	N	.045711	v4

UX displacement



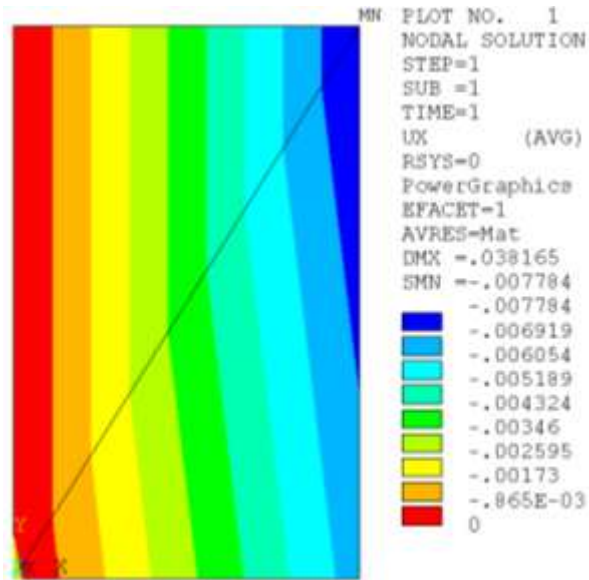
PLOT NO. 1
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UX (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.045711
 SMN =-.012724
 .012724
 .01131
 .009896
 .008483
 .007069
 .005655
 .004241
 .002828
 .001414
 0

UY displacement

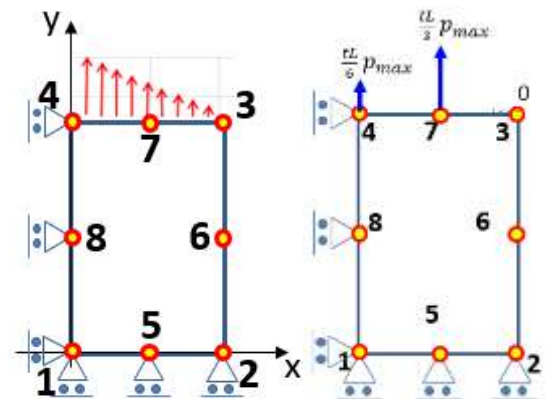
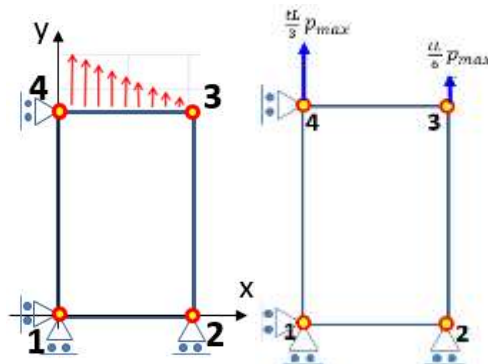
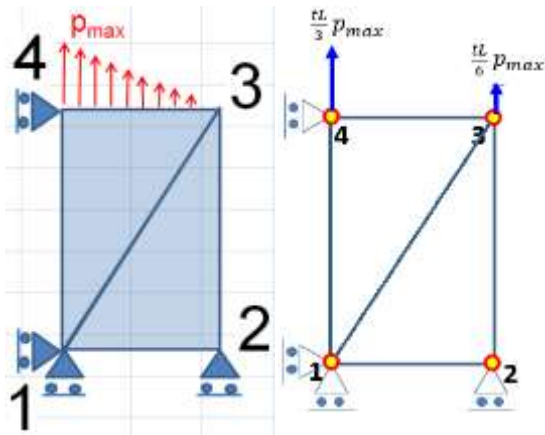
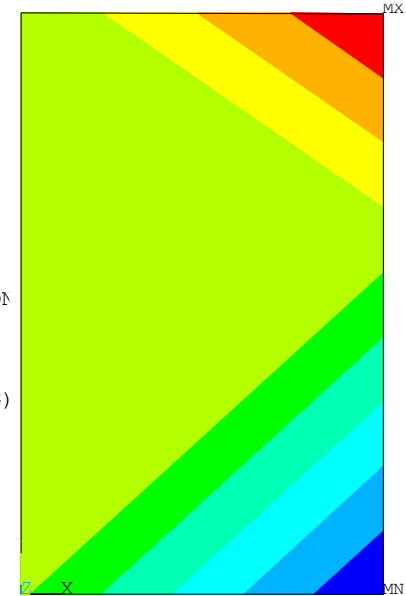
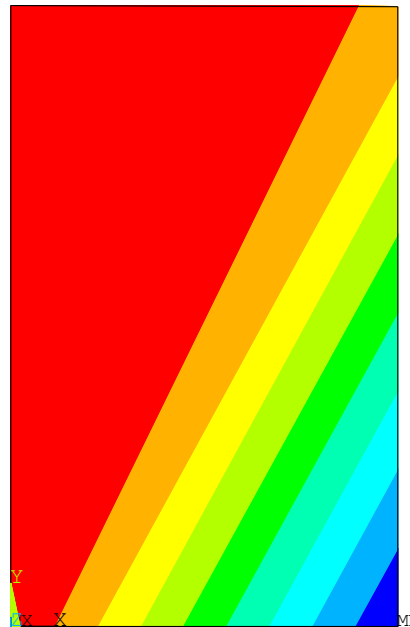


PLOT NO. 2
 NODAL SOLUTION
 STEP=1
 SUB =1
 TIME=1
 UY (AVG)
 RSYS=0
 PowerGraphics
 EFACET=1
 AVRES=Mat
 DMX =.045711
 SMX =.045711
 0
 .005079
 .010158
 .015237
 .020316
 .025395
 .030474
 .035553
 .040632
 .045711

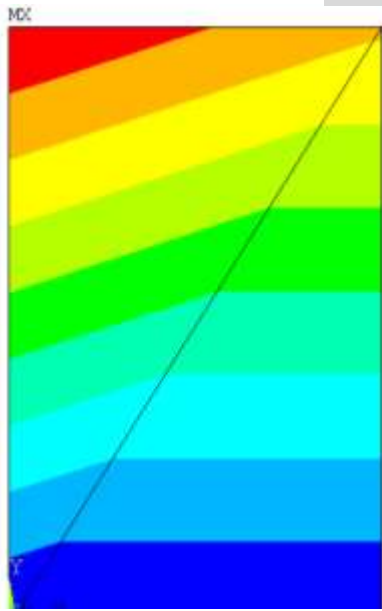
Displacements in X direction



UX displacement



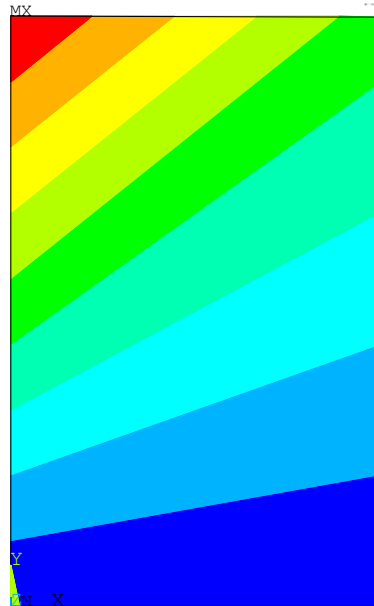
Displacements in Y direction



PLOT NO. 2
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.038165
SMX =.038165

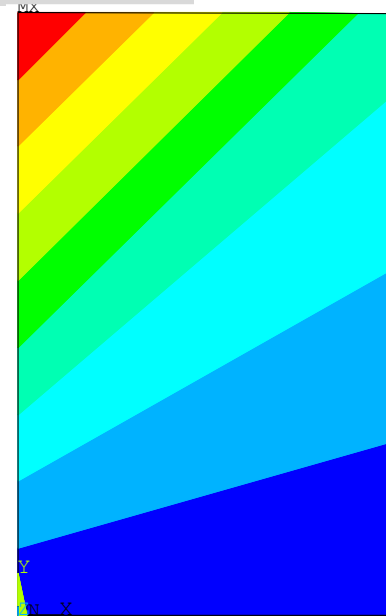
0
.004241
.008481
.012722
.016962
.021203
.025444
.029684
.033925
.038165

UY displacement



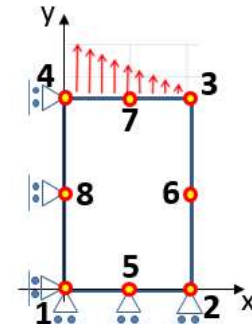
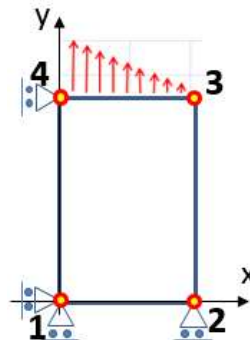
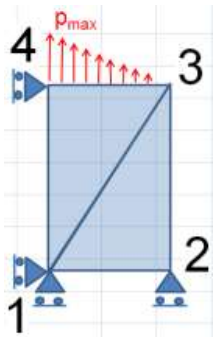
PLOT NO. 2
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.045711
SMX =.045711

0
.005079
.010158
.015237
.020316
.025395
.030474
.035553
.040632
.045711

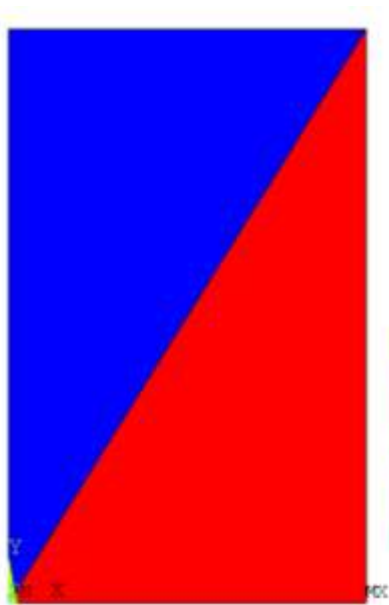


PLOT NO. 4
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.04694
SMX =.04694

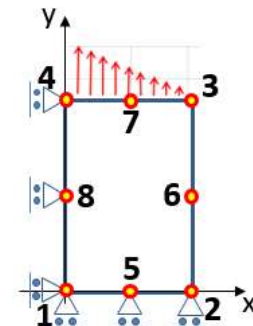
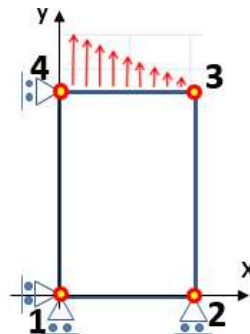
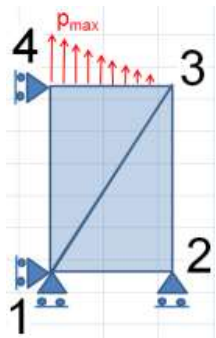
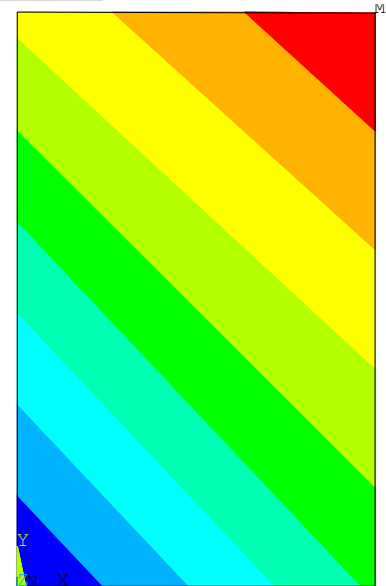
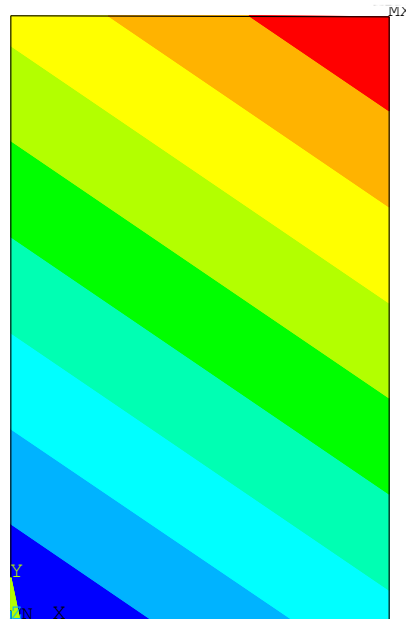
0
.005216
.010431
.015647
.020862
.026078
.031293
.036509
.041724
.04694



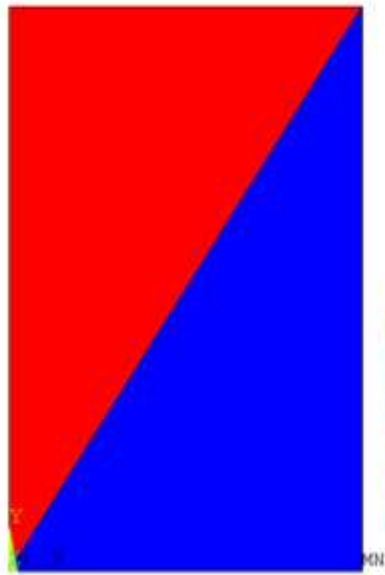
Strain in X direction



ϵ_x strain

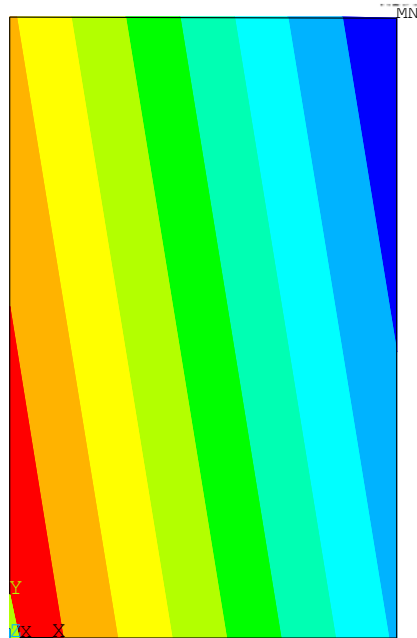


Strain in Y direction

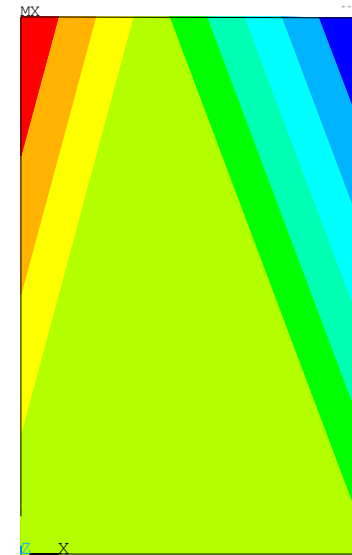


.380E-03
 .391E-03
 .402E-03
 .412E-03
 .423E-03
 .434E-03
 .445E-03
 .456E-03
 .466E-03
 .477E-03

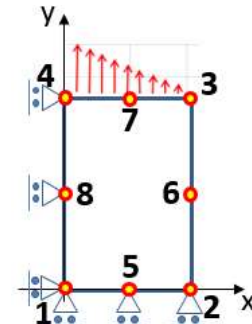
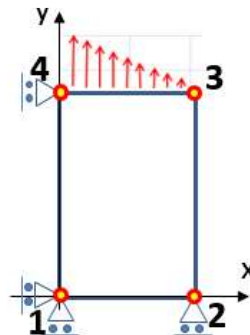
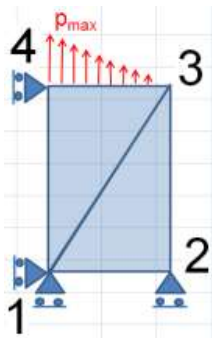
ϵ_y strain



.249E-03
 .289E-03
 .329E-03
 .369E-03
 .409E-03
 .449E-03
 .489E-03
 .529E-03
 .569E-03
 .609E-03

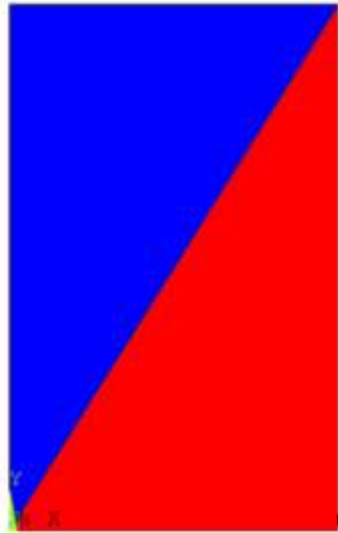


.238E-04
 .107E-03
 .189E-03
 .272E-03
 .355E-03
 .438E-03
 .520E-03
 .603E-03
 .686E-03
 .769E-03

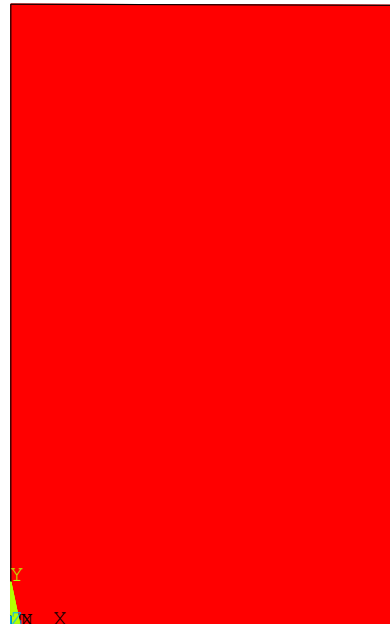


Shear strain

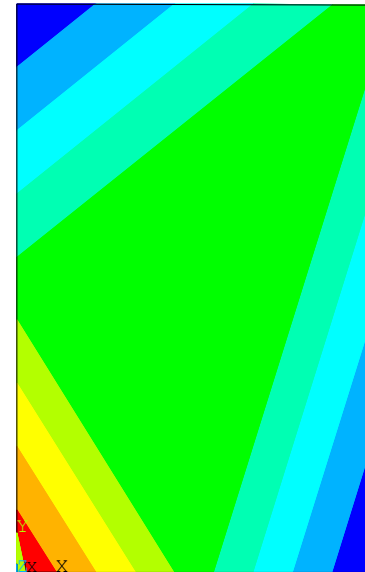
γ_{xy} strain



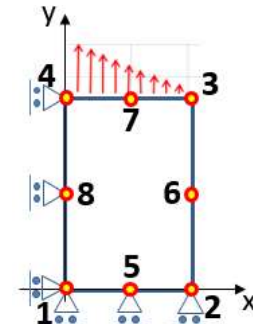
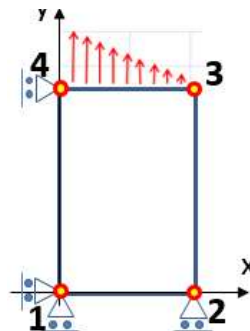
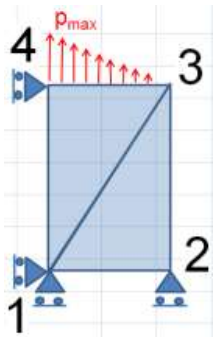
PLOT NO. 1
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
EPELXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =-.155E-03
SMX =-.160E-04
-.155E-03
-.140E-03
-.124E-03
-.109E-03
-.933E-04
-.779E-04
-.624E-04
-.470E-04
-.315E-04
-.160E-04



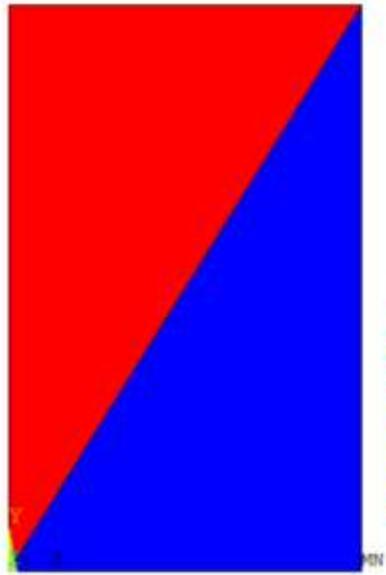
PLOT NO. 5
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
EPELXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.045711
SMN =-.159E-03
SMX =-.159E-03



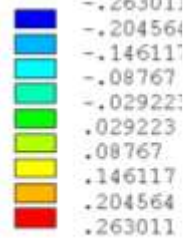
PLOT NO. 14
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
EPELXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =-.259E-03
SMX =.105E-03
-.259E-03
-.219E-03
-.178E-03
-.138E-03
-.974E-04
-.569E-04
-.164E-04
.241E-04
.646E-04
.105E-03



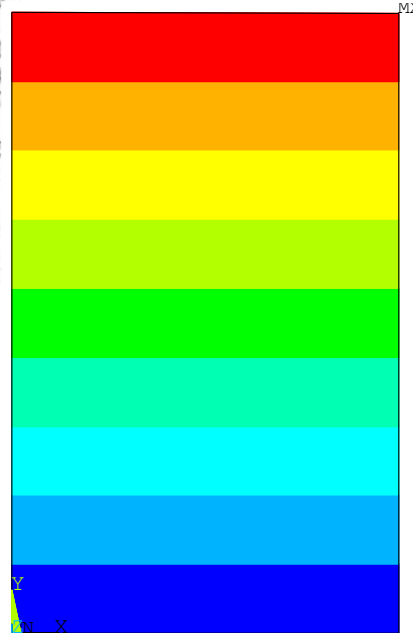
Stress in X direction



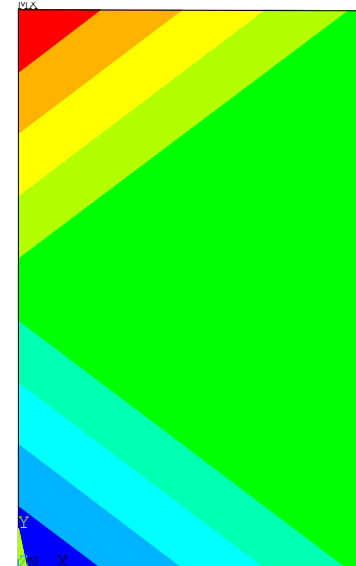
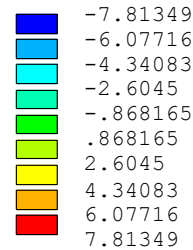
PLOT NO. 3
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.038165
SMN =-.263011
SMX =.263011



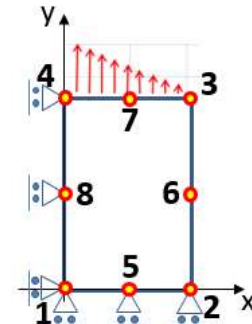
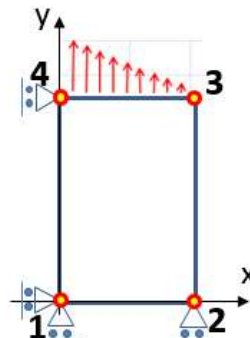
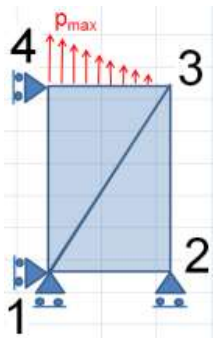
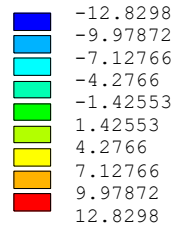
σ_x stress



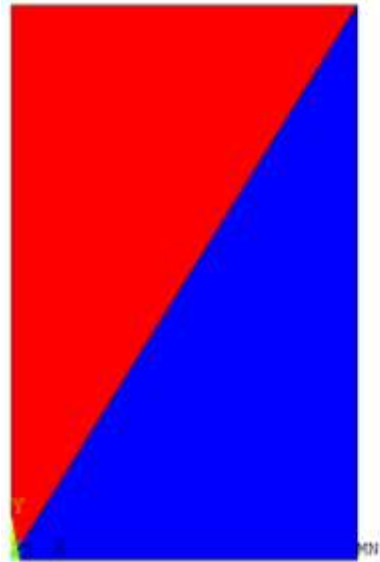
PLOT NO. 6
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =-7.81349
SMX =7.81349



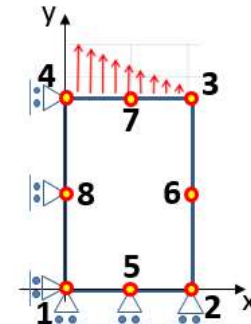
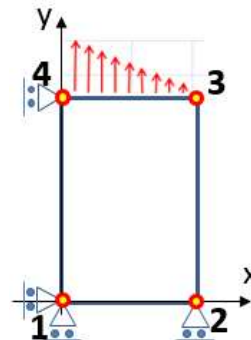
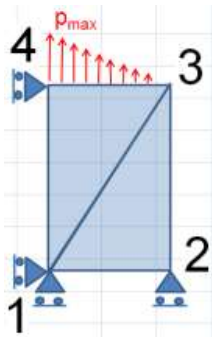
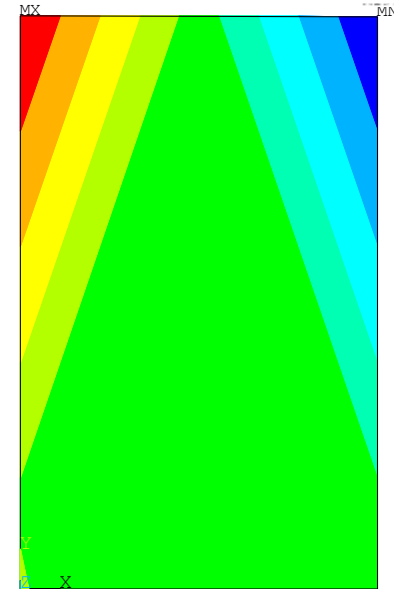
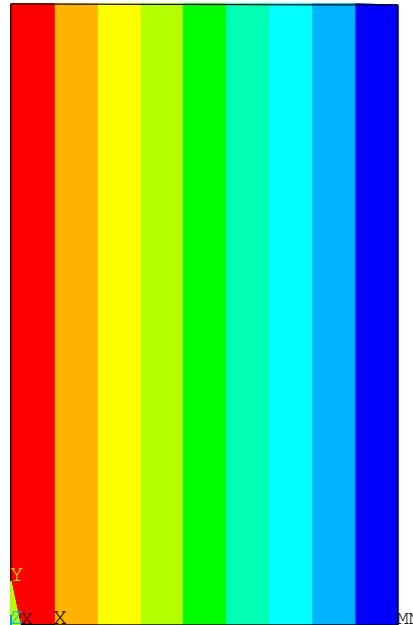
PLOT NO. 5
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.04694
SMN =-12.8298
SMX =12.8298



Stress in Y direction

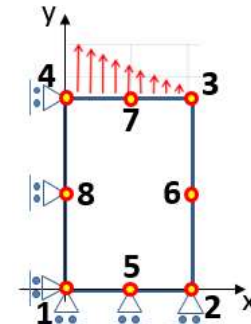
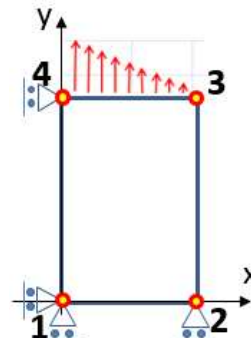
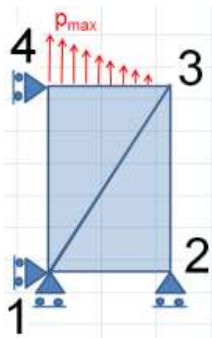
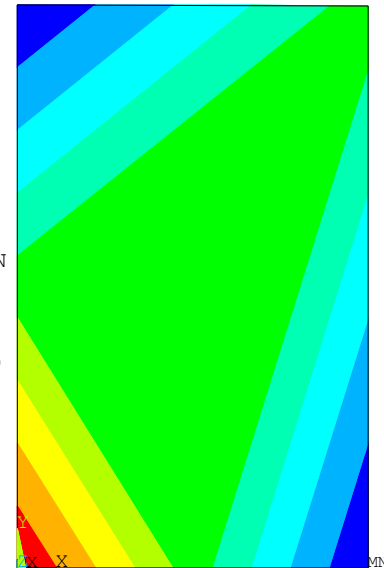
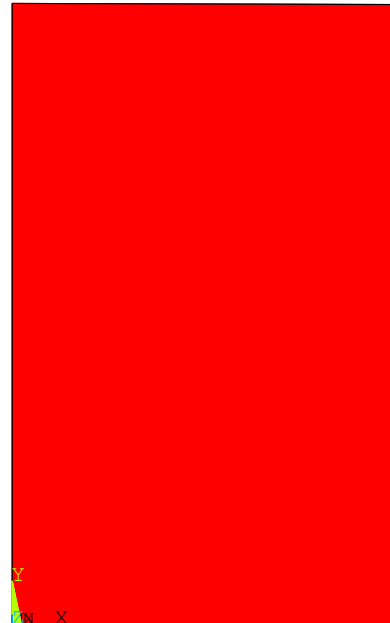
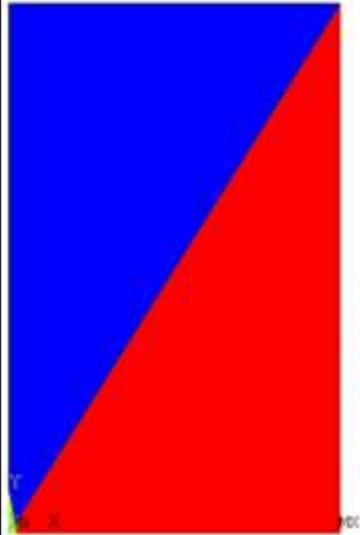


σ_y stress

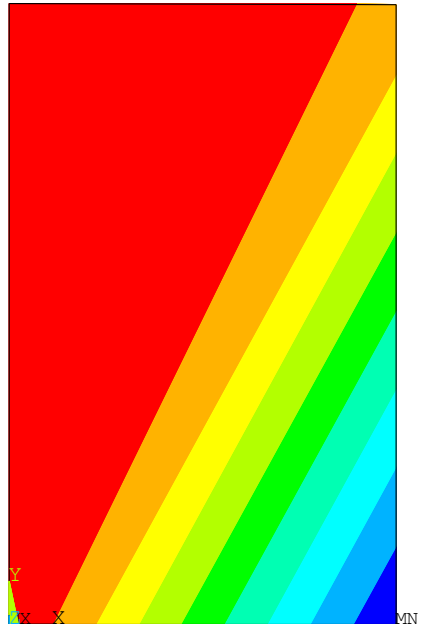
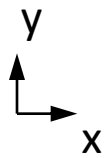


Shear stress

τ_{xy} stress

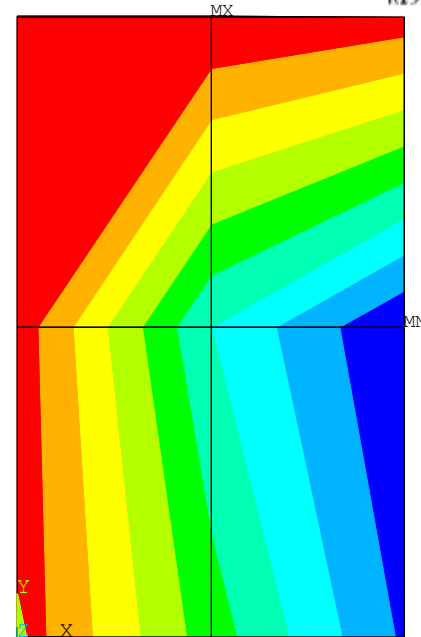


The impact of discretization on the quality of results (*4-node elements*)

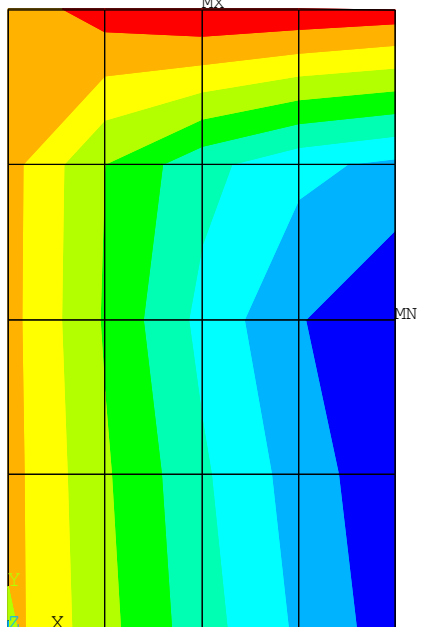


PLOT NO. 1
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.045711
SMN =-.012724
SMX =-.012724
-.01131
-.009896
-.008483
-.007069
-.005655
-.004241
-.002828
-.001414
0

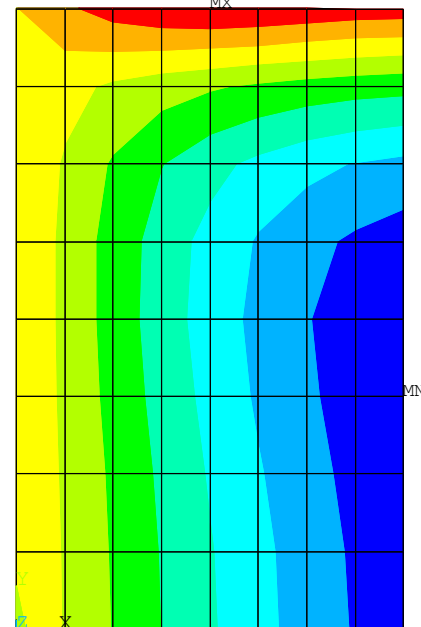
UX
[mm]



PLOT NO. 10
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046436
SMN =-.009823
SMX =.451E-03
-.009823
-.008682
-.00754
-.006398
-.005257
-.004115
-.002974
-.001832
-.690E-03
.451E-03

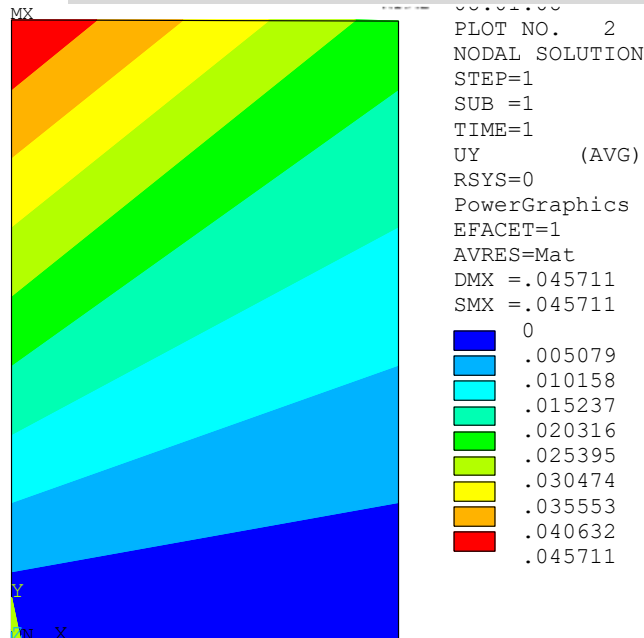
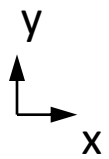


PLOT NO. 15
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046861
SMN =-.009151
SMX =.002032
-.009151
-.007909
-.006666
-.005424
-.004181
-.002938
-.001696
-.453E-03
.789E-03
.002032

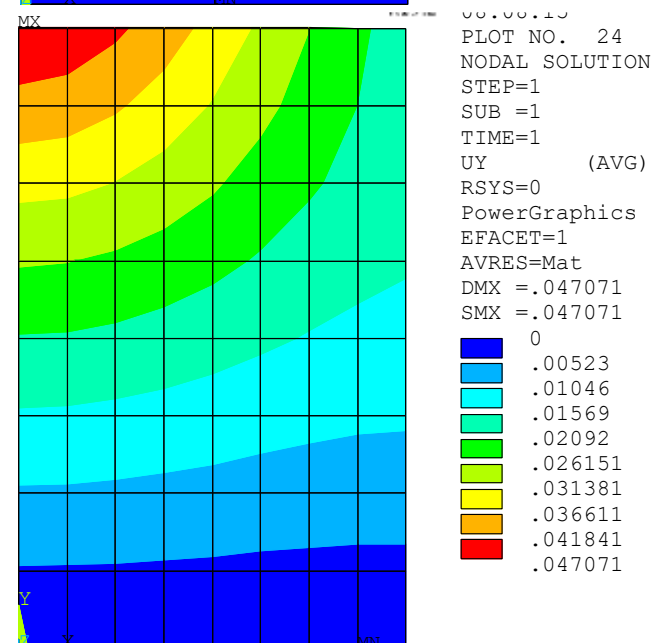
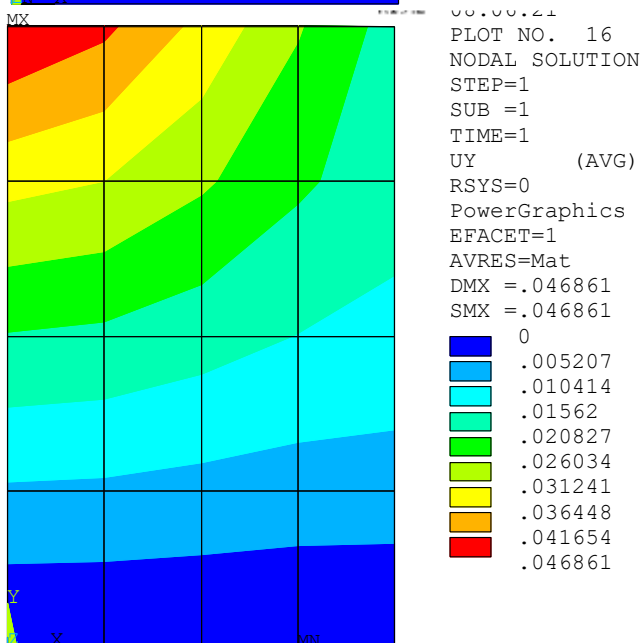
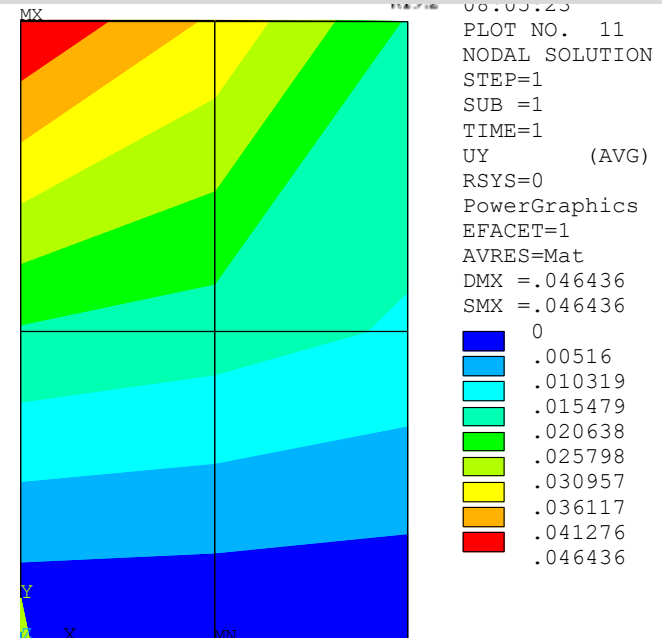


PLOT NO. 23
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
UX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047071
SMN =-.008957
SMX =.002617
-.008957
-.007671
-.006385
-.005099
-.003813
-.002527
-.001241
.452E-04
.001331
.002617

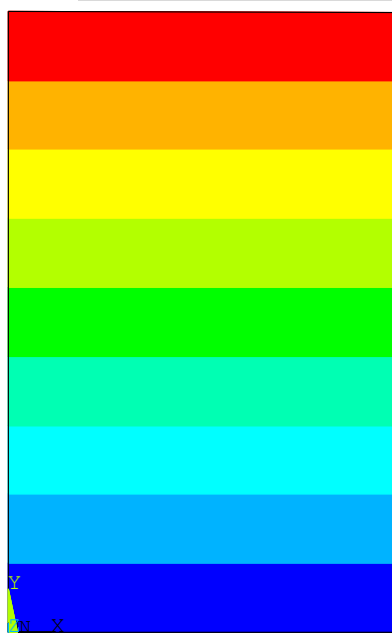
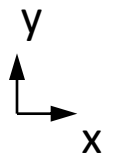
The impact of discretization on the quality of results (*4-node elements*)



UY
[mm]



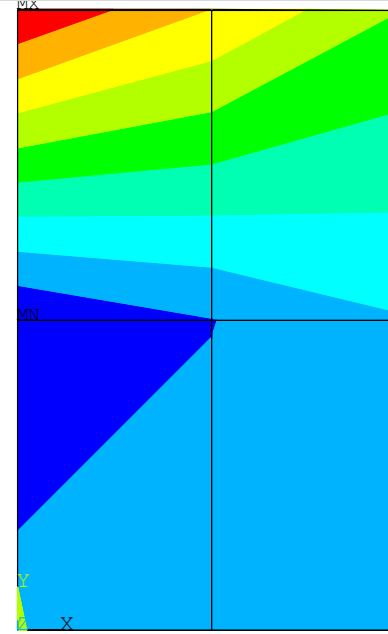
The impact of discretization on the quality of results (*4-node elements*)



PLOT NO. 6
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SX (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =-7.81349
SMX =7.81349

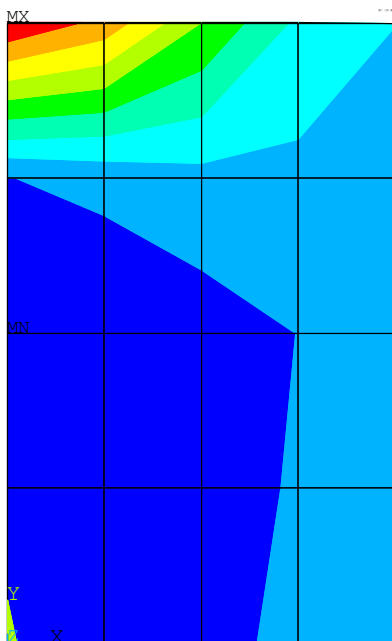
σ_x
[MPa]
**NODAL
SOLUTION**

-7.81349
-6.07716
-4.34083
-2.6045
-.868165
.868165
2.6045
4.34083
6.07716
7.81349



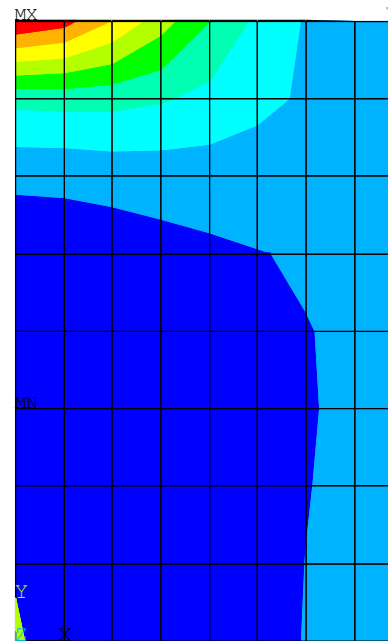
PLOT NO. 31
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046436
SMN =-5.95909
SMX =14.5388

-5.95909
-3.68154
-1.404
.873548
3.15109
5.42864
7.70619
9.98373
12.2613
14.5388



PLOT NO. 20
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046861
SMN =-4.63339
SMX =24.1699

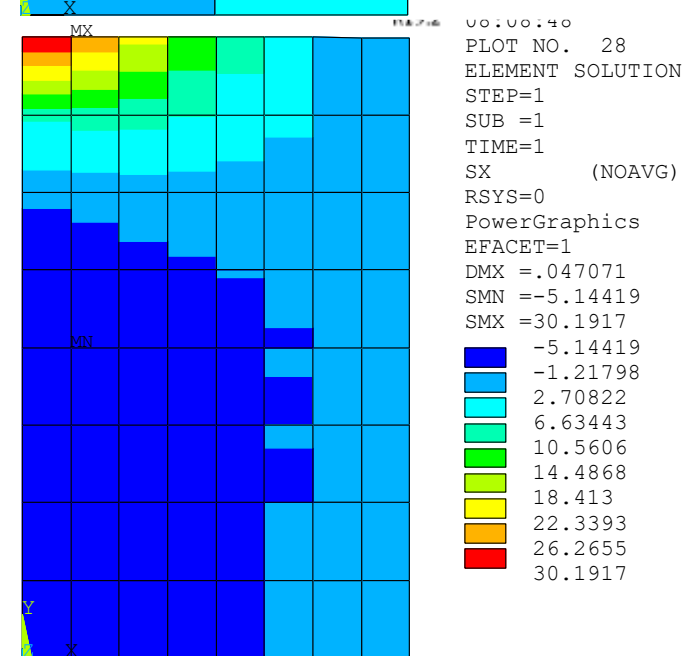
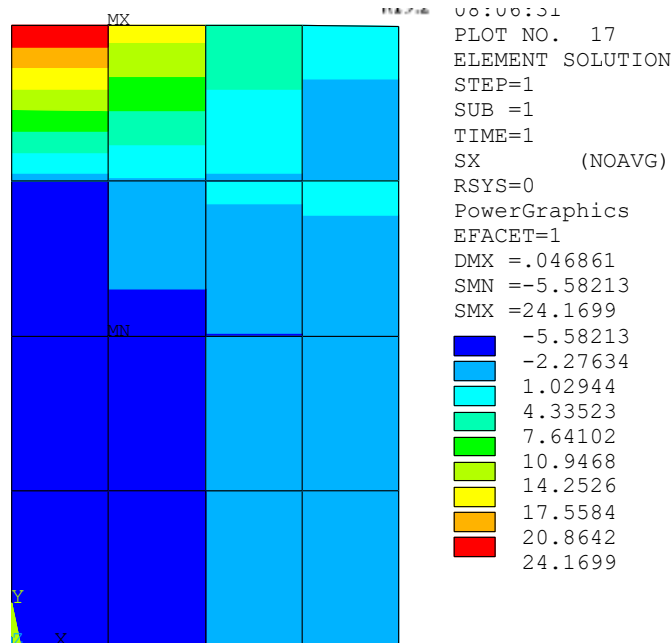
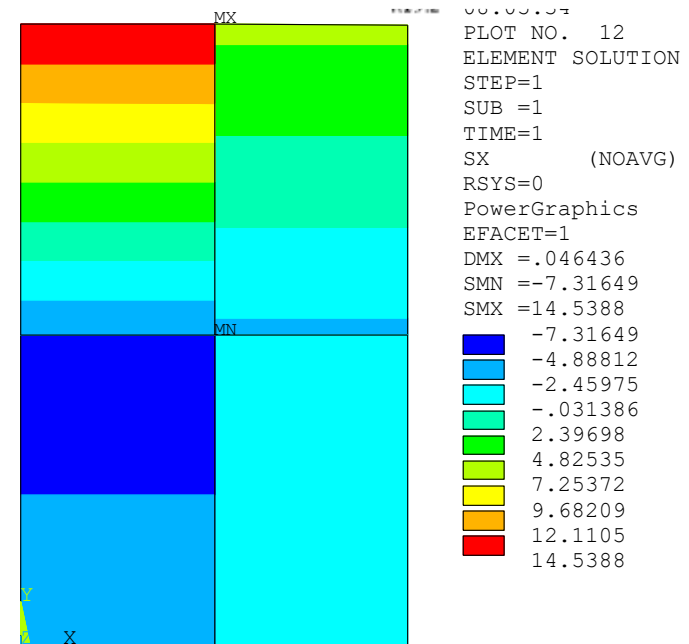
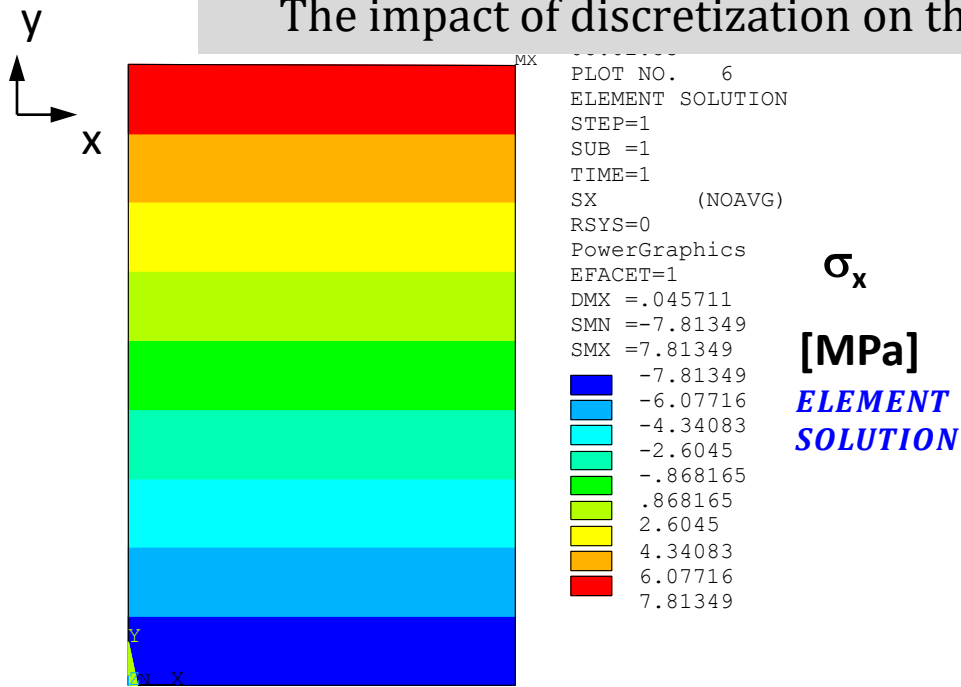
-4.63339
-1.43302
1.76735
4.96772
8.16809
11.3685
14.5688
17.7692
20.9696
24.1699



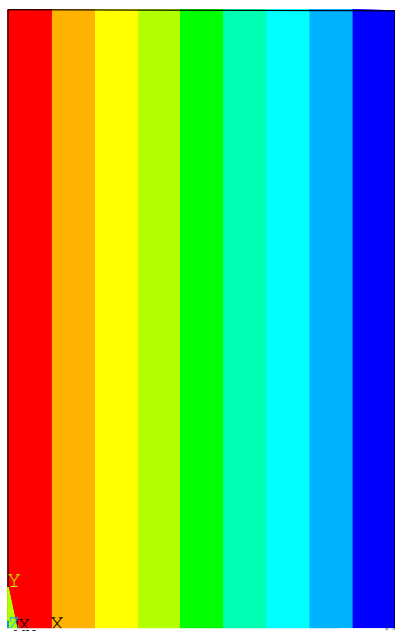
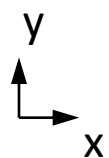
PLOT NO. 25
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SX (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047071
SMN =-4.69728
SMX =30.1917

-4.69728
-.820734
3.05582
6.93237
10.8089
14.6855
18.562
22.4386
26.3151
30.1917

The impact of discretization on the quality of results (*4-node elements*)



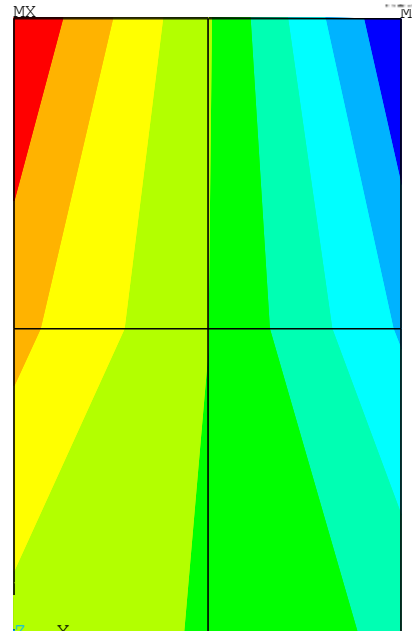
The impact of discretization on the quality of results (*4-node elements*)



PLOT NO. 7
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =20.0025
SMX =39.9975

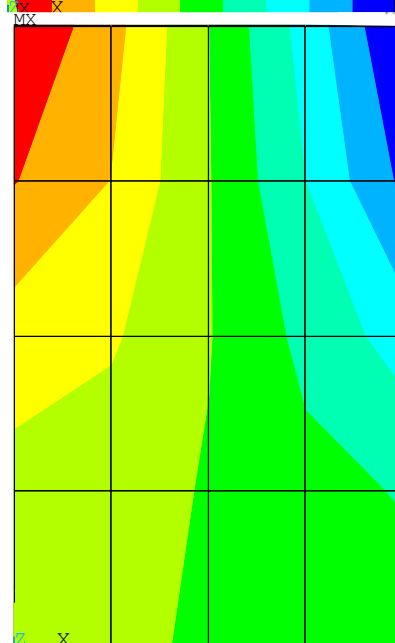
σ_y
[MPa]
**NODAL
SOLUTION**

20.0025
22.2242
24.4459
26.6675
28.8892
31.1108
33.3325
35.5541
37.7758
39.9975



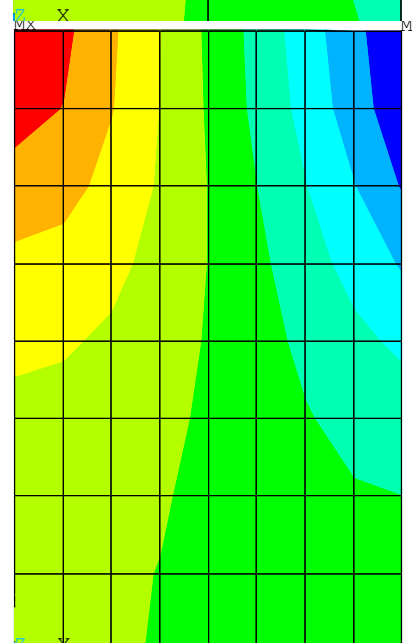
PLOT NO. 32
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046436
SMN =9.68398
SMX =47.7465

9.68398
13.9132
18.1423
22.3715
26.6007
30.8298
35.059
39.2882
43.5173
47.7465



PLOT NO. 21
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046861
SMN =3.04724
SMX =52.8834

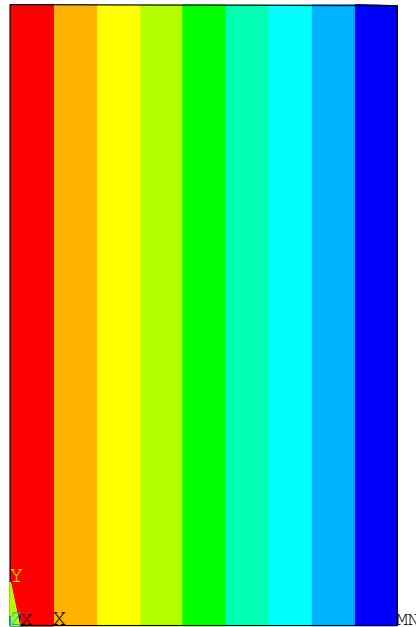
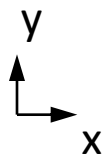
3.04724
8.5846
14.122
19.6593
25.1967
30.734
36.2714
41.8087
47.3461
52.8834



PLOT NO. 26
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047071
SMN =.427497
SMX =56.1359

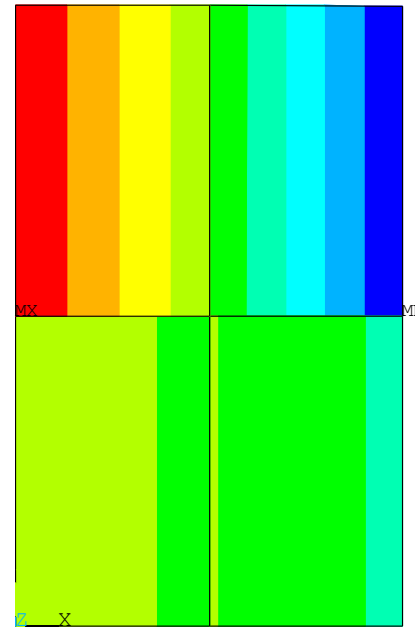
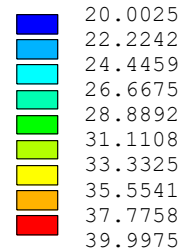
.427497
6.61732
12.8071
18.997
25.1868
31.3766
37.5664
43.7562
49.946
56.1359

The impact of discretization on the quality of results (*4-node elements*)

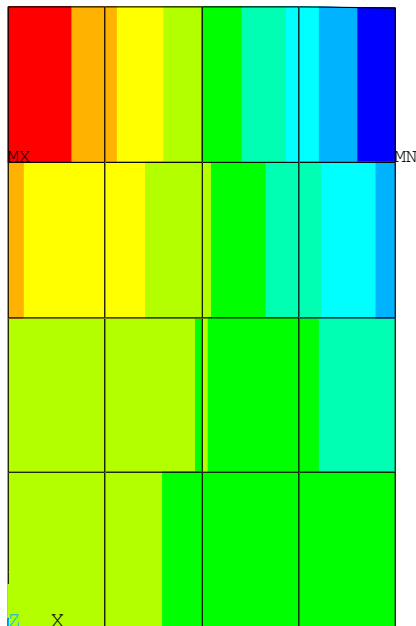
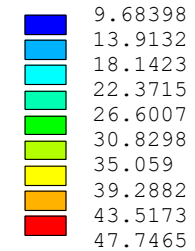


PLOT NO. 7
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =20.0025
SMX =39.9975

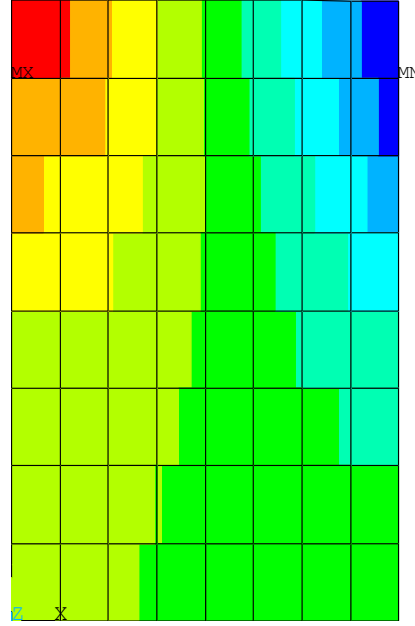
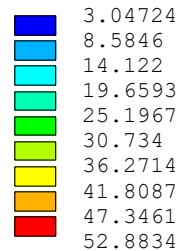
σ_y
[MPa]
*ELEMENT
SOLUTION*



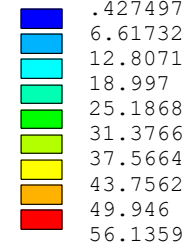
PLOT NO. 13
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.046436
SMN =9.68398
SMX =47.7465



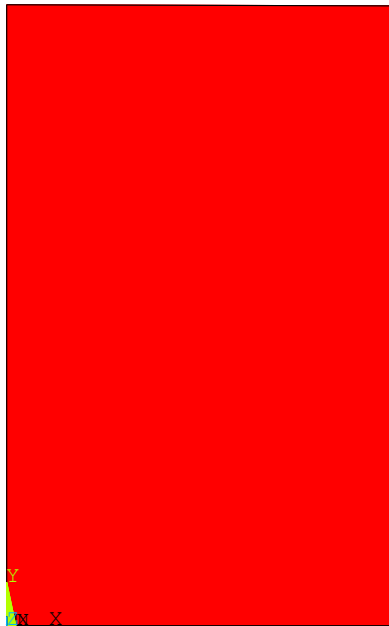
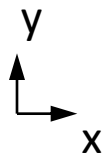
PLOT NO. 18
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.046861
SMN =3.04724
SMX =52.8834



PLOT NO. 29
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.047071
SMN =.427497
SMX =56.1359

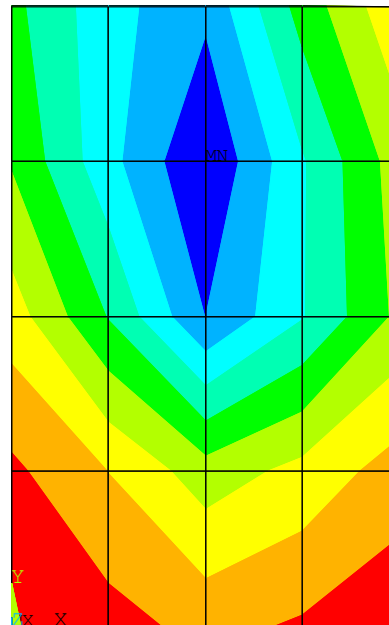


The impact of discretization on the quality of results (*4-node elements*)



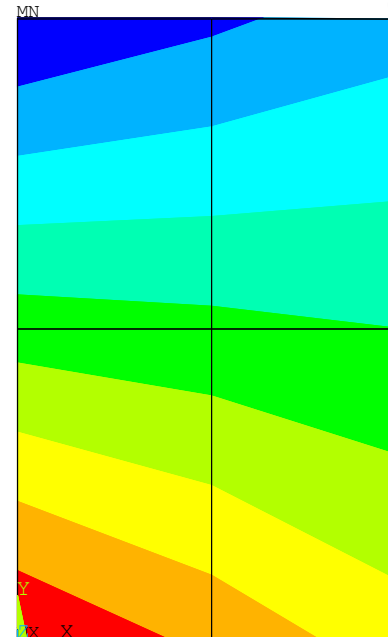
PLOT NO. 8
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =-4.16719
SMX =-4.16719

τ_{xy}
[MPa]
**NODAL
SOLUTION**



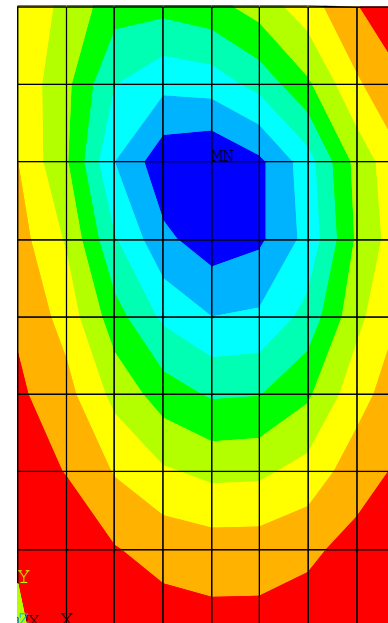
PLOT NO. 22
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046861
SMN =-5.49877
SMX =-.425279

Blue	-5.49877
Light Blue	-4.93504
Light Green	-4.37132
Green	-3.8076
Yellow-Green	-3.24388
Yellow	-2.68016
Orange	-2.11644
Red-Orange	-1.55272
Red	-.989
Dark Red	-.425279



PLOT NO. 33
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.046436
SMN =-4.70816
SMX =-1.65009

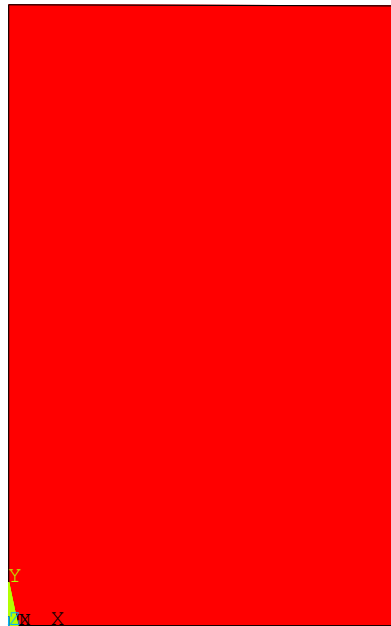
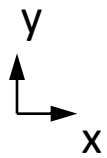
Blue	-4.70816
Light Blue	-4.36837
Light Green	-4.02859
Green	-3.6888
Yellow-Green	-3.34902
Yellow	-3.00923
Orange	-2.66944
Red-Orange	-2.32966
Red	-1.98987
Dark Red	-1.65009



PLOT NO. 27
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SXY (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.047071
SMN =-6.68
SMX =-.106963

Blue	-6.68
Light Blue	-5.94966
Light Green	-5.21933
Green	-4.48899
Yellow-Green	-3.75865
Yellow	-3.02831
Orange	-2.29798
Red-Orange	-1.56764
Red	-.837301
Dark Red	-.106963

The impact of discretization on the quality of results (*4-node elements*)

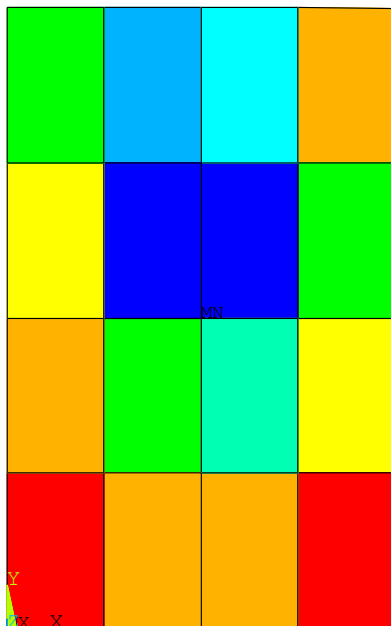


PLOT NO. 8
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.045711
SMN =-4.16719
SMX =-4.16719

τ_{xy}

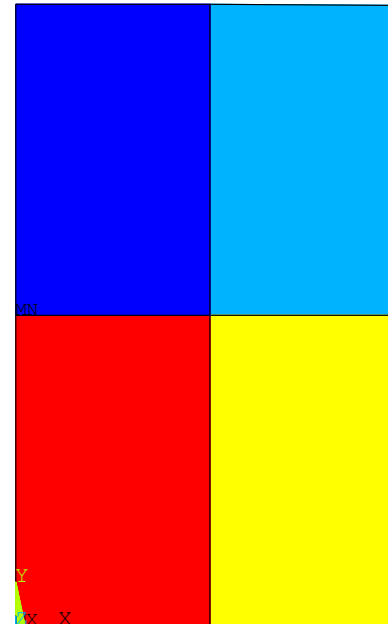
[MPa]

**ELEMENT
SOLUTION**



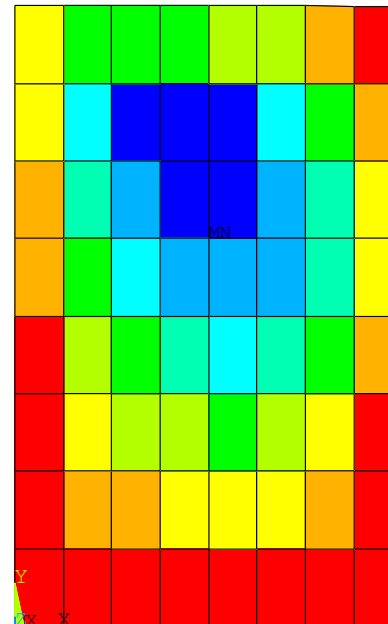
PLOT NO. 19
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.046861
SMN =-6.33246
SMX =-.425279

Legend for Plot 19:
-6.33246
-5.6761
-5.01975
-4.3634
-3.70704
-3.05069
-2.39434
-1.73799
-1.08163
-.425279



PLOT NO. 14
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.046436
SMN =-4.70816
SMX =-1.65009

Legend for Plot 14:
-4.70816
-4.36837
-4.02859
-3.6888
-3.34902
-3.00923
-2.66944
-2.32966
-1.98987
-1.65009



PLOT NO. 30
ELEMENT SOLUTION
STEP=1
SUB =1
TIME=1
SXY (NOAVG)
RSYS=0
PowerGraphics
EFACET=1
DMX =.047071
SMN =-6.83544
SMX =-.106963

Legend for Plot 30:
-6.83544
-6.08783
-5.34022
-4.59261
-3.84501
-3.0974
-2.34979
-1.60218
-.854572
-.106963